Indian Statistical Institute, Bangalore Centre
B.Math. (I Year) : 2012-2013
Semester II : Mid-Semestral Examination
Probability Theory II

04.03.2015 | Time: $2\frac{1}{2}$ hours. | Maximum Marks : 80

Note: Notation and terminology are understood to be as used in class. The paper carries 82 marks. Any score above 80 will be taken as 80. State clearly the results you are using in your answers.

1. (10 + 12 = 22 marks) Let $\lambda > 0$ be given, and C be a constant; suppose $f(\cdot,\cdot)$ is defined by

$$f(x,y) = \begin{cases} Ce^{-\lambda x}e^{-\lambda y}, & 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Find C so that f is a probability density function on $\mathbb{R}^2$.

(ii) Let (X, Y) be a two dimensional random variable with $f(\cdot,\cdot)$ as its probability density function. Find the marginal probability density functions. Are X and Y independent?

2. (20 marks) Let X and Y be independent random variables having respectively exponential distribution with parameter $\lambda > 0$, and uniform distribution over $(0, 1)$. Find the probability density function of $X + Y$.

3. (20 marks) Let X, Y be independent standard normal random variables. Let $W = 3X,\ Z = X-Y$. Show that (W, Z) is a two dimensional absolutely continuous random variable and find its probability density function.

4. (8 + 12 = 20 marks) (i) Let (X, Y) be a two dimensional absolutely continuous random variable such that X and Y have finite second moments. Show that $E(XY)$ exists.

(ii) Let $(X_1, X_2, \cdots, X_n)$ be an $n-$dimensional absolutely continuous random variable such that X_i has finite second moments for each $1 \leq i \leq n$. Show that $\text{Var}(X_1+X_2+\cdots+X_n)$ exists and find an appropriate expression for $\text{Var}(X_1 + X_2 + \cdots + X_n)$.